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On smoothness recognition over reals

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Let us consider generalized register machines over the field $(\mathbb{R}, 0, 1, +, -, \times, <)$ or over another real closed field [**1**]. They are closely related to the machines defined by L. Blum, M. Shub, and S. Smale (1989). Each register contains an element of $\mathbb{R}$. There exist index registers containing nonnegative integers. The running time is said polynomial, when the total number of operations performed before the machine halts is bounded by a polynomial in the number of registers occupied by the input. Initially, this number is placed in the zeroth index register. One can also define a nondeterministic generalized register machine that receives a few hints over $\mathbb{R}$.

Let polynomials over $\mathbb{R}$ be identified with sequences of their coefficients using some monomial order. For a positive integer k, the term "almost all k-tuples" means "all k-tuples but a set covered by a vanishing locus of a nonzero polynomial in k variables with integer coefficients". The generic computational complexity had been defined by I. Kapovich, A. G. Myasnikov, P. Schupp, and V. Shpilrain (2003) and extensively studied by A. N. Rybalov [**2**]. The machine never makes mistakes, but it can warn there is no way to accept or reject some input. These rare inputs are called vague. This concept is applicable over $\mathbb{R}$.

Definition. Let us consider a generalized register machine over $\mathbb{R}$ with three halting states: ACCEPT, REJECT, and VAGUE. The machine is said to be generic, when both conditions hold: (1) the machine halts on every input and (2) for every positive integer k and for almost all inputs that occupy exactly k registers, the machine does not halt at the VAGUE state.

Theorem. *There exists a generic generalized register machine that recognizes whether a given projective cubic hypersurface defined by a form over $\mathbb{R}$ of the type $x_0^3 + \cdots + x_n^3 + (\alpha_0 x_0 + \cdots + \alpha_n x_n)^3$ is smooth at every real point of the intersection with a given projective hyperplane defined by a linear form of the type $x_0 + \beta x_n$, where $\beta \in \mathbb{R}$. The running time of the machine is polynomial in n.*

Remark. The same method can be applied to the smoothness recognition of some other cubic hypersurfaces. Moreover, it is easy to check whether there exists a real singular point by a nondeterministic generalized register machine over $\mathbb{R}$. But in the hardest case, it seems impossible to check smoothness in deterministic polynomial time. Thus, some hard problems can be solved in generic polynomial time.

References

[1] Neumann E., Pauly A. A topological view on algebraic computation models. Journal of Complexity 44, 1–22 (2018). https://doi.org/10.1016/j.jco.2017.08.003

[2] Rybalov A. N. Generic amplification of recursively enumerable sets. Algebra and Logic 57:4, 289–294 (2018). https://doi.org/10.1007/s10469-018-9500-y

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