2. Y. Amice, "Interpolation p-adique," Bull. Soc. Math. France, 92, 117-180 (1964).
3. Yu. I. Manin, p-Adic Automorphic Functions [in Russian], Vol. 3, Current Problems of Mathematics, Moscow (1974).
4. M. Lazard, "Les zeros des fonctions analytiques d'une variable sur un corps value complet," Publs IHES, 14, 47-75 (1962).

A CONSEQUENCE OF THE MARTIN AXIOM

V. G. Kanovei

1. Introduction. Let us recall some well-known definitions.

Let P with the ordering $\leqslant$ be a partially ordered set. Two arbitrary elements $p, q \in P$ are said to be compatible if $\exists r\,[r \leqslant p$ and $r \leqslant q]$, and incompatible in the contrary case [1, p. 48]. A set $Q \subseteq P$ is called an antichain if any two different elements $p, q \in Q$ are incompatible. Let us call the following condition the countable antichain condition (in [1, p. 65] it is called the countable chain condition): Every antichain $Q \subseteq P$ is countable. Further, a set $D \subseteq P$ is said to be dense in P if for each $p \in P$ there exists a $q \in D$ such that $q \leqslant p$ [1, p. 49]. Let F be a family of subsets of the set P. A set $G \subseteq P$ is said to be F-generic if the following three conditions are fulfilled [1, p. 99]:

a) If $p \in P$, $q \in G$, and $q \leqslant p$, then $p \in G$.

b) If $p, q \in G$, then there exists an $r \in G$ such that $r \leqslant p$ and $r \leqslant q$.

c) If $D \in F$ is dense in P, then $D \cap G \neq 0$.

Finally, we will denote the cardinality 2^ω of the continuum by c.

The Martin axiom (MA) can be formulated in the following manner [1, p. 99]:

If P with the ordering $\leqslant$ is a partially ordered set that satisfies the countable antichain condition and if F is a family of subsets of P such that card (F) < c, then there exists an F-generic subset D of P.

See [2] for more details about this interesting axiom.

Further, for each set x the class of all sets, constrictible from x, is denoted by L[x] [1, p. 39]. Moreover, if α is an ordinal, then we will denote the α-th (according to counting) transfinite cardinal in L[x] by $\omega_\alpha^{L[x]}$ (the counting is started from zero, i.e., $\omega_0^{L[x]} = \omega$ for arbitrary x).

We will call the following statement the Levi axiom (LA): $(\forall x \subseteq \omega)$ [the ordinal $\omega_1^{L[x]}$ is countable in the universe of all sets].

The proposed name is stipulated by the fact that the model ZFC + LA was constructed and studied by Levi in [3]. In particular, the equiconsistency of the theories ZFC + LA and ZFC + "the axiom that there exists an inaccessible cardinal" is proved in [3].

And now we give the last group of definitions. Let $\varkappa$ be an ordinal. A set $A \subseteq \varkappa$ is said to be closed and unbounded in $\varkappa$ if the following two conditions are fulfilled: 1) $\bigcup (A \cap \alpha) \in A$ for each $\alpha \in \varkappa$ and 2) $\bigcup A = \varkappa$. A cardinal $\varkappa$ is called a Mahlo cardinal if it is inaccessible and each closed unbounded subset A of $\varkappa$ contains an inaccessible cardinal (see [4, p. 94].

The following theorem is proved in the present note.

THEOREM 1.1 (ZFC). Let MA, LA, and the relation c > ω_1 (i.e., the negation of CH) be fulfilled. Then ω_1 is a Mahlo cardinal in L[x] for each $x \subseteq \omega$.

The following corollary follows immediately from the above theorem and the definition of Mahlo cardinal.

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1) $s' \subseteq s$ and $t' \subseteq t$;

2) $s \cap y = s' \cap y$ for each $y \in t'$.

Thus, larger components of the set $p \in P$ correspond to a smaller (in the sense of this ordering) set p.

We interrupt the proof of the theorem for proving the following lemma.

LEMMA 3.3. P satisfies the countable antichain condition.

Proof of the Lemma. Let us observe that arbitrary $(s, t) \in P$ and $(s, t') \in P$ (with the same first components) are compatible, since the set $(s, t \cup t')$ obviously belongs to P and satisfies the relations $(s, t \cup t') \leqslant (s, t)$ and $(s, t \cup t') \leqslant (s, t')$. Therefore, incompatible p, $q \in P$ must have different first components. But the elements of the set P have a countable number of possible first components. The lemma is now obvious.

We now continue with the proof of Theorem 3.2. For each $y \in Y$ we set, by definition, $D_y = \{(s, t) \in P\colon y \in t\}$. For all $x \in X - Y$ and $n \in \omega$ we set, by definition, $D_{xn} = \{(s, t) \in P\colon$ there exists a $k \in x \cap s$ such that $k \geqslant n\}$. We prove two lemmas about denseness.

LEMMA 3.4. If $y \in Y$, then the set D_y is dense in P.

Proof. If $(s, t) \in P$, then $(s, t \cup \{y\}) \in D_y$ and $(s, t \cup \{y\}) \leqslant (s, t)$.

LEMMA 3.5. If $x \in X - Y$ and $n \in \omega$, then the set D_{xn} is dense in P.

Proof. Let $p = (s, t) \in P$. Let us construct a $q \in D_{xn}$ such that $q \leqslant p$. We observe that $x \notin Y$, and $t \subseteq Y$, and therefore, $x \notin t$. On the other hand, it follows from $x \in X \subseteq \mathcal{F}$ and $t \subseteq Y \subseteq X \subseteq \mathcal{F}$ that $t \cup \{x\} \subseteq \mathcal{F}$. Therefore, by Lemma 3.1, the set $x \cap y$ is finite for each $y \in t$. Hence there exists an $m \in \omega$ such that $m \geqslant n$ and the following condition is fulfilled:
(1) $(\forall k \geqslant m)\ (\forall y \in t)\ [k \notin x \cap y]$.

Further, it follows from the relation $x \in X \subseteq \mathcal{F}$ and Lemma 3.1 that x is infinite. Consequently, there exists a $k \in x$ such that $k \geqslant m$. Then $k \geqslant n$, and therefore the set q = $(s \cup \{k\}, t)$ belongs to D_{xn}.

It remains to verify the inequality $q \leqslant p$. Let us assume the contrary: $q \nleqslant p$. Since p = (s, t), this means that there exists a $y \in t$ such that $(s \cup \{k\}) \cap y \neq s \cap y$. It is clear that the last statement implies that $k \in y$. But $k \geqslant m$ and $k \in x$. We have obtained a contradiction with (1), which proves the relation $q \leqslant p$ and the lemma.

We continue with proof of Theorem 3.2. Let us set, by definition, $F = \{D_y\colon y \in Y\} \cup \{D_{xn}\colon x \in X - Y\ n \in \omega\}$. Since card $(X) \leqslant \omega_1$, it follows that card $(F) \leqslant \omega_1$. Therefore, taking Lemma 3.3 and the relation $c > \omega_1$ into account, we can apply MA and find an F-generic set $G \subseteq P$. We show that the set $z = \bigcup \{s\colon$ there exists a t such that $(s, t) \in G\}$ is the desired one in the sense of Theorem 3.2.

Verification of (i). Let $y \in Y$. We will prove that $z \cap y$ is finite. Since the set G is F-generic, by Lemma 3.4 we have $G \cap D_y \neq 0$. Let p = (s, t) belong to $D_y \cap G$. We will prove that $z \cap y \subseteq s$; since s is finite, by definition $(s, t) \in P$; therefore, this will be sufficient.

Let us assume the contrary: $k \in z \cap y$, but $k \notin s$. By the definition of z there exists a $q = (s', t') \in G$ such that $k \in s'$. Further, since p and q are elements of the set G, it follows from the definition of a generic set, Sec. 1, that there exists an $r = (s'', t'') \in P$ such that $r \leqslant p$ and $r \leqslant q$. By the definition of $\leqslant$ we have:

(1) $s' \subseteq s''$, and by the same token $k \in s''$, since $k \in s'$;

(2) $s \cap y = s'' \cap y$, since $y \in t$.

But these two statements give a contradiction since $k \in y$ and $k \notin s$. This contradiction completes the verification of (i).

Verification of (ii). Let $x \in X - Y$. We will show that $z \cap x$ is infinite. It is sufficient to verify that for each $n \in \omega$ there exists a $k \in z \cap x$ such that $k \geqslant n$.

Let $n \in \omega$. It follows from Lemma 3.5 and the F-genericity of G that $D_{xn} \cap G \neq 0$. Let $(s, t) \in D_{xn} \cap G$. Then $s \subseteq z$ by the definition of z, and there exists a $k \in s \cap x$ such that $k \geqslant n$ by the definition of D_{xn}. Thus, we have found a $k \in z \cap x$ such that $k \geqslant n$, which was required.

(ii) is verified for z, and Theorem 3.2 is proved.

4. Proof of Theorem 1.1. In this section we will assume that MA, LA, and inequality $c > \omega_1$ are fulfilled.

4.1. We will prove that $\Omega = \omega_1$ is a Mahlo cardinal in L[x] for each $x \in R$.

Let us assume the contrary: $u \in R$ is such that Ω is not a Mahlo cardinal in L[u]. Since Ω is inaccessible in L[u] by Lemma 2.1, by Proposition 2.2 this assumptions implies the existence of an unbounded closed subset A of Ω, $A \in L[u]$, that does not contain regular (in the sense of L[u]) cardinals.

We will obtain a contradiction from this assumption. The sets u and A of the indicated form are fixed in the following arguments. For each $\alpha \in \Omega$ we denote the α-th (according to magnitude) element of the set A by a_α (since A is an unbounded closed subset of $\Omega = \omega_1$ and we have assumed the axiom of choice, it follows that card (A) $= \Omega$). Without loss of generality we may assume that $a_0 = 0$.

THEOREM 4.2. There exists a sequence $d = (x_\alpha \colon \alpha \in \Omega)$ of elements of the set R such that the following conditions are fulfilled:

(a) $x_\alpha \notin \{x_\gamma \colon \gamma \in \alpha\}$ for any $\alpha \in \Omega$ and $x_0 = u$.

(b) $a_\alpha < \omega_1^{L[x_\alpha]}$ for each $\alpha \in \Omega$.

(c) $\Omega = \omega_1^{L[d]}$.

(d) If $\alpha \in \Omega$ is a limit ordinal, then x_α is the smallest (in the sense of the canonical complete ordering of the class $L[d|\alpha]$) $y \in R \cap L[d|\alpha]$. such that $y \notin \{x_\gamma \colon \gamma \in \alpha\}$ and $L[d|\alpha] = L[y]$.

(Comments on the statement (d): $d|\alpha$ is the restriction of the sequence d to α, i.e., $d|\alpha = (x_\gamma \colon \gamma \in \alpha)$.)

Proof. We construct the desired sequence by induction on α. The induction consists of three steps.

1*. We set $x_0 = u$.

2*. Let $\alpha \in \Omega$ and $x_\gamma \in R$ be constructed for all $\gamma \leqslant \alpha$. We now indicate the construction of $x_{\alpha+1}$. Since $a_{\alpha+1} \in A \subseteq \Omega$, the ordinal $a_{\alpha+1}$ is countable. Therefore, there exists a $y \in R$ such that $a_{\alpha+1} < \omega_1^{L[y]}$ and $y \notin \{x_\gamma \colon \gamma \leqslant \alpha\}$. We set $x_{\alpha+1}$ equal to one of these y.

3*. Let us assume that $\alpha \in \Omega$ is a limit ordinal and the "initial segment" $d|\alpha = (x_\gamma \colon \gamma \in \alpha)$ has already been constructed such that (b) is fulfilled for all $\gamma < \alpha$. We indicate the construction of x_α.

We first prove that $a_\alpha < \omega_1^{L[d|\alpha]}$ Indeed, by the assumptions made by us, $a_\gamma < \omega_1^{L[x_\gamma]}$ for all $\gamma \in \alpha$. All the more, $a_\gamma < \omega_1^{L[d|\alpha]}$. But the set A is closed in Ω, and this implies that $a_\alpha \leqslant \omega_1^{L[d|\alpha]}$ since α is a limit ordinal.

We will now prove that the equality $a_\alpha = \omega_1^{L[d|\alpha]}$ is impossible. Indeed, the cardinal $\omega_1^{L[d|\alpha]}$ is regular in $L[d|\alpha]$ and, all the more, regular in L[u], since $u = x_0 \in L[d|\alpha]$. On the other hand, a_α is not a regular cardinal in L[u] by virtue of the relation $a_\alpha \in A$ and the choice of A and u. Thus, the indicated equality is indeed impossible. Together with the above-proved relation $a_\alpha \leqslant \omega_1^{L[d|\alpha]}$ we finally have $a_\alpha < \omega_1^{L[d|\alpha]}$.

All the more we then have $\alpha < \omega_1^{L[d|\alpha]}$, i.e., the statement "$d|\alpha = (x_\gamma \colon \gamma \in \alpha)$ is a sequence of elements of the set R of countable length α" is true in $L[d|\alpha]$. Therefore there exists a $y \in R \cap L[d|\alpha]$ such that $y \notin \{x_\gamma \colon \gamma \in \alpha\}$ and $L[d|\alpha] = L[y]$. We set x_α equal to the smallest (in the sense of the canonical complete ordering of the class $L[d|\alpha]$) among all these y. The construction of $(x_\alpha \colon \alpha \in \Omega)$ is complete.

Let us verify the conditions (a)-(d). The condition (a) is fulfilled obviously for $\alpha = 0$ and is clearly fulfilled for $\alpha > 0$ on account of the constructions 2* and 3*. The condition (b) follows for $\alpha = 0$ from the assumption $a_\alpha = 0$ in 4.1. The condition (b) for limit ordinals α follows immediately from the construction 2*.

Let us prove (b) in the case of a limit ordinal $\alpha \in \Omega$. By the construction 3* we have $L[x_\alpha] = L[d|\alpha]$. We also have $a_\alpha < \omega_1^{L[d|\alpha]}$(see the arguments of 3*). Combining these two statements, we get (b).

Further, (c) is obvious from (b), and (d) follows immediately from the construction 3*. Thus, $d = (x_\alpha\colon \alpha \in \Omega)$ is the desired sequence. The theorem is proved.

In the sequel the sequence $d = (x_\alpha\colon \alpha \in \Omega)$ with the properties (a)-(d) is fixed.

4.3. Let us observe that if $\alpha \in \Omega$ is a limit ordinal, then, having the "initial segment" $d|\alpha = (x_\gamma\colon \gamma \in \alpha)$, we uniquely restore x_α in $L[d|\alpha]$ with the help of 4.2 (d). The aim of the following lemma is to chose a $z \in R$ that helps us to do the same for nonlimit ordinals α. Before stating the lemma let us introduce the "convolution" $x * n = \{2^n(2k+1) - 1\colon k \in x\}$ for $x \in R$ and $n \in \omega$.

LEMMA 4.3. There exists a $z \in R$ such that $x_{\alpha+1} = \{n\colon$ the set $z \cap S(x_\alpha * n)$ is finite$\}$ for each $\alpha \in \Omega$.

The proof is based on Theorem 3.2. We set, by definition $X = \{S(x_\alpha * n)\colon \alpha \in \Omega$ and $n \in \omega\}$ and $Y = \{S(x_\alpha * n)\colon \alpha \in \Omega$ and $n \in x_{\alpha+1}\}$. Before applying 3.2 we prove two auxiliary propositions.

(1) If $x_\alpha * n = x_\beta * m$, then $\alpha = \beta$ and $m = n$.

Indeed, the indicated equality obviously implies that $m = n$ and $x_\alpha = x_\beta$ by the definition of the convolution $*$. But the statement that if $\alpha \neq \beta$, then $x_\alpha \neq x_\beta$ follows from 4.2 (a). Now (1) is obvious.

(2) If $S(x_\alpha * n) \in Y$, then $n \in x_{\alpha+1}$.

Indeed, by the definition of the set Y it follows that there exist $\beta \in \Omega$ and $m \in \omega$ such that $S(x_\alpha * n) = S(x_\beta * m)$ and $m \in x_{\beta+1}$. But the equality $S(x) = S(y)$ implies that $x = y$ by 3.1. Consequently, $x_\alpha * n = x_\beta * m$. Hence, applying (1), we have $\alpha = \beta$ and $m = n$. Now (2) follows from the relation $m \in x_{\beta+1}$.

We now return to the proof of the lemma. It follows from the definition of the sets X and Y that $Y \subseteq X \subseteq \mathfrak{F}$ and $\operatorname{card}(X) = \Omega = \omega_1$. Therefore, by Theorem 3.2 there exists a $z \in R$ such that: (3) $x \in Y$ if and only if $z \cap x$ is finite, for each $x \in X$

It now follows from statements (2) and (3) and the definition of the set Y that the set z is the desired one. The lemma is proved.

We fix the set $z \in R$ whose existence has been asserted in the above lemma. The following theorem is the decisive moment in the proof of Theorem 1.1.

THEOREM 4.4. The sequence $d = (x_\alpha\colon \alpha \in \Omega)$ belongs to L[u, z].

Proof. We indicate the following procedure for the computation of the set $x'_\alpha \in R$ in L[u, z].

1**. We set $x'_0 = u$.

2**. If $x'_\alpha \in R$ has been constructed, then $x'_{\alpha+1} = \{n\colon z \cap S(x_\alpha * n)$ is finite$\}$.

3**. If $\alpha \in \Omega$ is a limit ordinal and the "initial segment" $d' = (x'_\gamma\colon \gamma \in \alpha)$ has already been constructed, then x'_α is the smallest (in the sense of the canonical complete ordering of the class L[d']) among all $y \in R \cap L[d']$ such that $L[d'] = L[y]$ and $y \notin \{x'_\gamma\colon \gamma \in \alpha\}$.

By induction over α it is easily proved that $x'_\alpha = x_\alpha$ for all $\alpha \in \Omega$. Namely, the desired equality is obvious for $\alpha = 0$: $x'_\alpha = x_\alpha = u$. The induction step $\alpha \to \alpha + 1$ is considered with regard for the choice of z (satisfying the condition of Lemma 4.3). The induction step for a limit ordinal α is considered with regard for 4.2 (d). Thus $x'_\alpha = x_\alpha$ for all $\alpha \in \Omega$.

Further, the above construction is valid in L[u, z] also. By the same token $d = (x_\alpha\colon \alpha \in \Omega)$ belongs to L[u, z]. The theorem is proved.

COROLLARY 4.5. There exists a $y \in R$ such that $\omega_1^{L[y]} = \Omega$.

Proof. Indeed, we set, by definition, $y = \{2n\colon n \in u\} \cup \{2n+1\colon n \in z\}$. Then $u, z \in L[y]$, and therefore $d \in L[y]$ by Theorem 4.4. Using 4.2 (c), we get the desired result.

4.6. We now complete the proof of Theorem 1.1. Above in 4.1 we have assumed the contrary. This led us to the existence of a $y \in R$ such that $\omega_1^{L[y]} = \Omega = \omega_1$. By the same token we get a contradiction with the assumption (made in accordance with the statement of Theorem 1.1) that LA is fulfilled. This contradiction refutes the contradictory assumption made in 4.1 and completes the proof of Theorem 1.1.

LITERATURE CITED

1. T. J. Jech, "Lectures in set theory with particular emphasis on the method of forcing," Lect. Notes Math., 217 (1971).
2. D. Martin and R. Solovay, "Internal Cohen extensions," Ann. Math. Logic, 2, No. 2, 143-178 (1970).
3. A. Levy, "Definability in axiomatic set theory. II," in: Mathematical Logic and Foundations of Set Theory, North-Holland, Amsterdam (1970), pp. 129-145.
4. K. Devlin, "Indescribability properties and small large cardinals," in: G. H. Muller, A. Oberschelp, and K. Potthoff, Logic Conference, Kiel, 1974, Lect. Notes Math., Vol. 499, Springer-Verlag, Berlin–New York (1975), pp. 89-114.
5. R. B. Jensen and R. M. Solovay, "Some applications of almost disjoint sets," in: Mathematical Logic and Foundations of Set Theory, North-Holland, Amsterdam (1970), pp. 84-103.

RIGIDITY OF CONVEX SURFACES WITH ISOLATED BOUNDARY

V. T. Fomenko

Let S_m be an m-connected surface with the boundary ∂S_m in the three-dimensional Euclidean space E_3. We say that the surface S_m has an isolated boundary ∂S_m if for an arbitrary infinitesimal deformation of the surface the variation of the normal curvature of the boundary is equal to zero. Under the assumption that $S_m \in D_{3,p}$, $p > 2$; $\partial S_m \in C^{1,\lambda}$, $0 < \lambda < 1$, $m = 1, 2, 3$, and the Gaussian curvature K is positive up to the boundary, the rigidity of the surface with isolated boundary was proved in 1952 by I. N. Vekua. The condition $K \geqslant k_0 > 0$, k_0 = const, characterizes local convexity of the surface S_m. In 1947 N. V. Efimov proved the rigidity of surfaces with isolated plane boundary and conjectured that an arbitrary convex surface with isolated boundary is rigid. The justification for this conjecture for regular ovaloids S_m, $m \geqslant 1$, with smooth holes of a sufficiently general form was given in 1964 by M. I. Voitsekhovskii. The condition of smoothness (and not piecewise smoothness) of the boundary is essential here: There exist convex surfaces with piecewise-smooth boundary admitting nontrivial infinitesimal deformations that preserve curvature of the boundary. In the present note Voitsekhovskii's result is carried over to piecewise regular convex surfaces with smooth boundary in the form of the following theorem.

THEOREM 1. Let F_m be a convex surface glued from surfaces of the class C^3 of nonnegative Gaussian curvature K that do not contain plane regions, where the lines of gluing L_i ($i = 1, 2, \ldots, n$) are simple closed or nonclosed nonintersecting curves of the class C^2. Let the surface F_m be bounded by the curves l_i ($i = 1, 2, \ldots, m$) of the class C^2 that do not intersect the lines of gluing, where the curves l_i ($i = 1, 2, \ldots, m$) do not contain straight-line segments and lie on convex cones with vertex at a certain elliptic point 0 of the surface F_m. Then the surface F_m with isolated boundary is rigid.

The theorem is proved by the method of integral formulas with the help of appropriate results of [1-3].

1. Following [3], we take the origin of the Cartesian system of coordinates (x, y, z) at the point 0 of the surface F_m, and dispose the surface F_m in the half plane $z \geqslant 0$ so that the plane z = 0 becomes tangent to the surface at the point 0. We carry out the projective transformation of the space; $x' = x/z$, $y' = y/z$, and $z' = 1/z$, under which the surface F_m transforms into a surface F_m single-valuedly projecting on the plane $z' = 0$, whose radius

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